Two-Echelon Capacitated Vehicle Routing Problem

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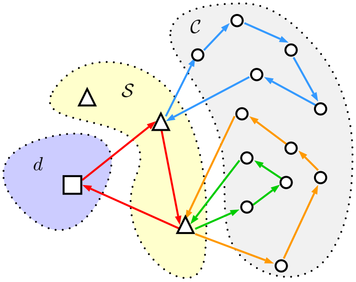


Figure: Example of 2ECVRP solution

- Cover the demand for freight of a set of customers
- Freight stored in a distribution center
- Urban trucks ship freight from the distribution center to satellites
- City freighters deliver freight to customers

Objective : Minimize the total transportation cost respecting partitioning, capacity and transfer constraints.

Motivations [Crainic et al., 2009]

Goal : Deliver freight to customers located in city centers

but freight transportation :

- competes with people transportation for the capacity of the streets
- contributes to congestion
- participates in environmental nuisances (noise and pollution)

and freight transportation grows because of :

- distribution practices based on low inventories and timely deliveries
- electronic commerce
- urban migration

Motivations - Freight transportation in the urban area of Bordeaux

[Agence d'urbanisme Bordeaux Aquitaine, 2019]

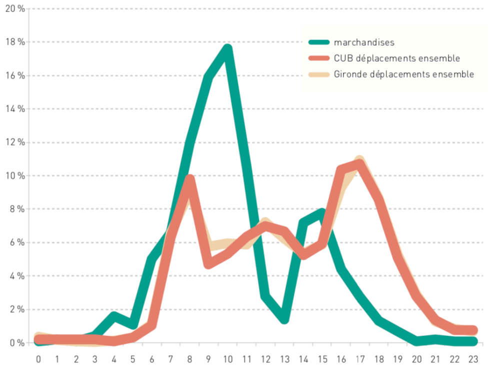


Figure: Hourly distribution of freight and people movements in 2013 (picture from p.9 of reference

- 40% of enterprises located in Bordeaux

Freight vs people transportation

- 53% of freight movements between 7-10am
- High use of illegal parking in Bordeaux
  - 59% bikes, 44% cars, 60% vans, 51% trucks

19 years evolution

- Less trucks ($\geq$ 3.5t)
  - $1/2$ movements with trucks in 1994
  - $< 1/3$ movements with trucks in 2013

Literature

Survey by [Cuda et al., 2015]

Exact approaches

- Branch and cut algorithm [Perboli et al., 2011] [Jepsen et al., 2013]
  - Flow-based formulation
- Branch and cut and price [Santos et al., 2015]
- Bounding procedures, selection of first-level solutions and resolution of several MDCVRP [Baldacci et al., 2013]
  - Path-based formulations [Baldacci et al., 2013] [Santos et al., 2015]
    - First-level route, second-level route
    - Quantity of freight delivered by a first-level route to a satellite

Literature

Survey by [Cuda et al., 2015]

Exact approaches

- Best results by [Baldacci et al., 2013]
- No instances with more than 6 satellites solved

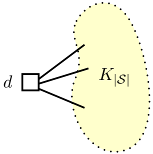
Heuristic approaches

- GRASP [Crainic et al., 2013]
- Adaptive large neighborhood search heuristic [Hemmelmayr et al., 2012]
- Large neighbourhood based heuristic [Breunig et al., 2016]
- Tours generated by neighbourhood search + solution improved by a MIP (best results) [Wang et al., 2017] [Amarouche et al., 2018]
- Solutions of good quality for instances up to 10 satellites and 200 customers

Subproblems

First level $G_1 = (V_1, E_1)$

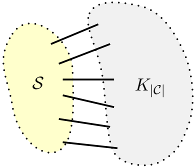
- $V_1 = \{d\} \cup \mathcal{S}$



- routes $p \in P$ enumerated (at most 10 satellites $=$ 1023 routes)
- $\tilde{x}^p \in \{0, 1, 2\}^{|E_1|}$ – characteristic vector of route p
- $\lambda_p =$ number of trucks using route p

Second level $G_2 = (V_2, E_2)$

- $V_2 = \mathcal{S} \cup \mathcal{C}$



- routes $r \in R$ generated (RCSP)
- $\tilde{z}^r \in \{0, 1, 2\}^{|E_2|}$ – characteristic vector of route r
- $\mu_r =$ number of freighters using route r

2nd-echelon subproblems description

- One subproblem for each satellite $s \in \mathcal{S}$
- Let z_{ij}^s be the number of times edge (i, j) is used by a path started at satellite s

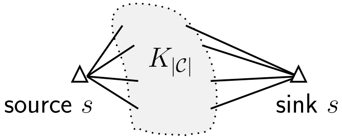


Figure: Graph for generating routes in R_s

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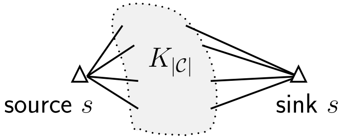
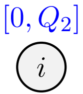


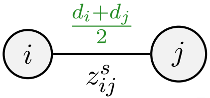
Figure: Graph for generating routes in R_s

- Capacity is the only one resource.

Resource bounds at node i



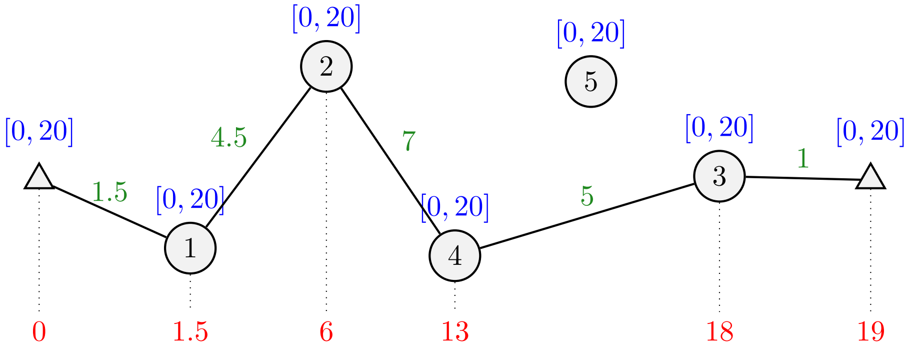
Resource consumption on edge (i, j)



2nd-echelon subproblems description

Example of a solution

Example with $Q_2 = 20$, $d_1 = 3$, $d_2 = 6$, $d_3 = 2$, $d_4 = 8$, and $d_5 = 4$.



- Capacity bounds at nodes
- Resource consumption on edge
- Accumulated capacity consumption

Master formulation (Path-based formulation)

- λ_p = nb trucks using first-level route $p \in P$
- μ_r = 1 if a city freighter uses second-level route $r \in R$
- $y_e^s = \sum_{r \in R_s} \tilde{z}_e^r \mu_r$ (map to edge e of 2nd-echelon subproblem of satellite s)

$$
\begin{aligned}
\min \quad & \sum_{p \in P} \sum_{e \in E_1} f_e^T \tilde{x}_e^p \lambda_p + \sum_{s \in \mathcal{S}} \sum_{e \in E_2} f_e^T y_e^s + \sum_{p \in P} \sum_{s \in \mathcal{S}} f_s^H w_s^p \\
\text{s.t.} \quad & \sum_{s \in \mathcal{S}} \sum_{e \in \delta(c)} y_e^s = 2 \qquad c \in \mathcal{C} \\
& \sum_{p \in P} \lambda_p \leq |\mathcal{K}| \\
& \sum_{r \in R_s} \mu_r \leq L_s \qquad s \in \mathcal{S} \\
& \sum_{r \in R} \mu_r \leq |\mathcal{L}| \\
& \lambda_p, \mu_r \in \mathbb{N} \qquad p \in P, r \in R
\end{aligned}
$$

Master formulation

Link between the two levels

- w_s^p quantity of freight delivered by route p to satellite s.

$$
\begin{aligned}
w_s^p &\le Q_1 \delta_s^p \lambda_p && p \in P, s \in \mathcal{S} \\
\sum_{s \in \mathcal{S}} w_s^p &\le Q_1 \lambda_p && p \in P \\
d_s &\le \sum_{p \in P} w_s^p && s \in \mathcal{S} \\
w_s^p &\ge 0 && p \in P, s \in \mathcal{S}
\end{aligned}
$$

with

- $\delta_s^p = \frac{1}{2} \sum_{e \in \delta(s)} \tilde{x}_e^p = 1$ iff route p visits satellite s.
- $d_s = \sum_{c \in \mathcal{C}} d_c \frac{1}{2} \sum_{e \in \delta(c)} y_e^s$ demand supplied from satellite s.

Freight flow variables vanish

Let u_S be the number of urban trucks visiting subset S of satellites.

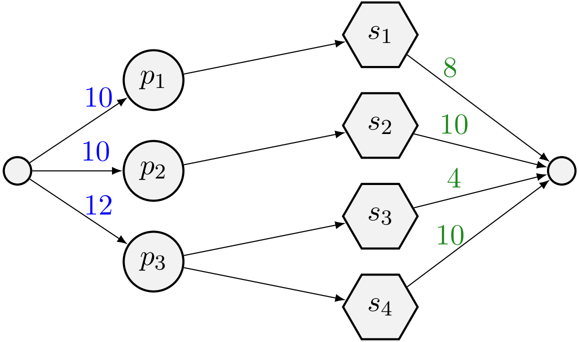
$$Q_1 u_S \geq \sum_{s \in S} d_s \qquad S \subseteq \mathcal{S}$$

$Q_1 = 12$

$\lambda_{p_1} = \frac{5}{6}$

$\lambda_{p_2} = \frac{5}{6}$

$\lambda_{p_3} = 1$



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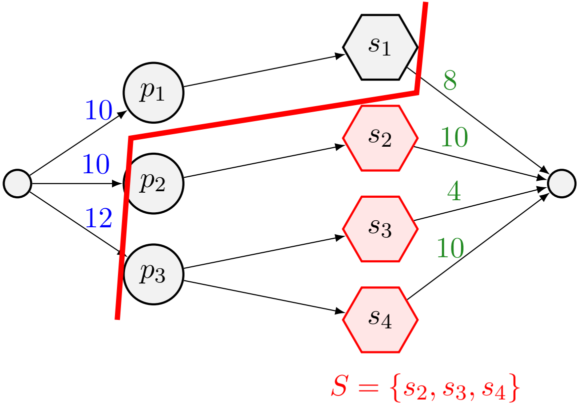

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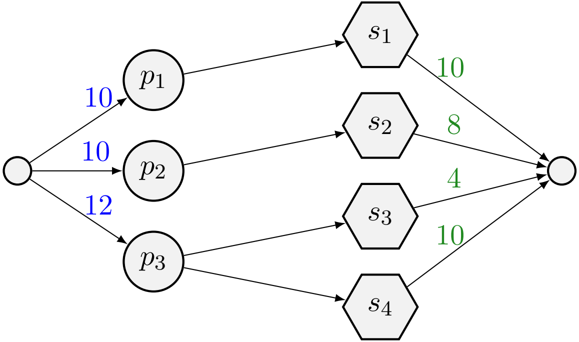

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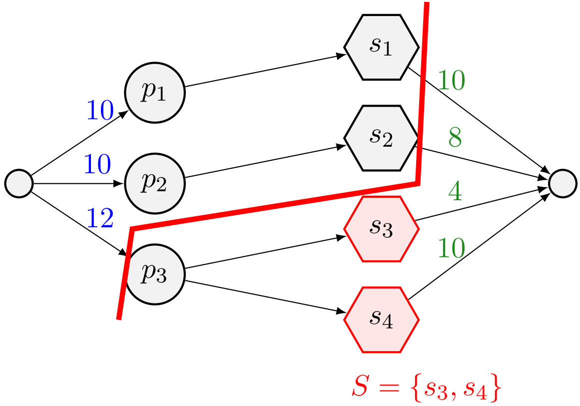
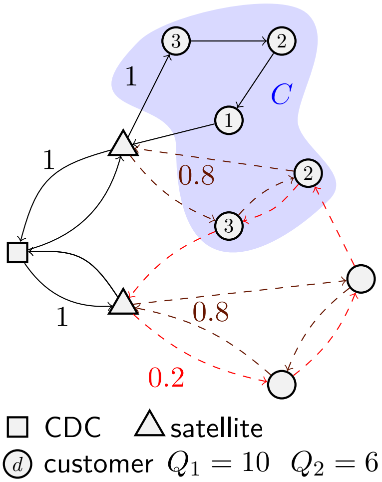


Satellites Supply Inequalities (SSI)



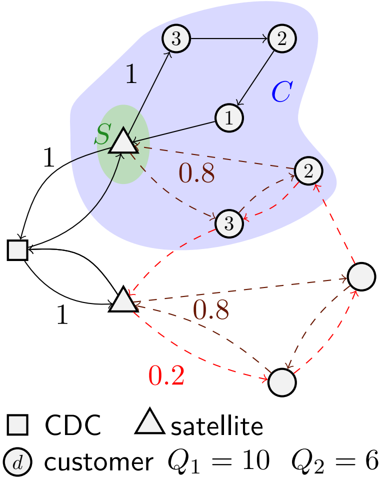
- variable y_e^s : nb of freighters from $s \in \mathcal{S}$ using $e \in E_2$
- data d_C : total demand of customers in $C \subset \mathcal{C}$

Rounded Capacity Cut

$$\sum_{s \in \mathcal{S}} \sum_{e \in \delta(C)} y_e^s \geq 2 \left\lceil \frac{d_C}{Q_2} \right\rceil$$

In the example : $4 \geq 2 \left\lceil \frac{11}{6} \right\rceil$

Satellites Supply Inequalities (SSI)



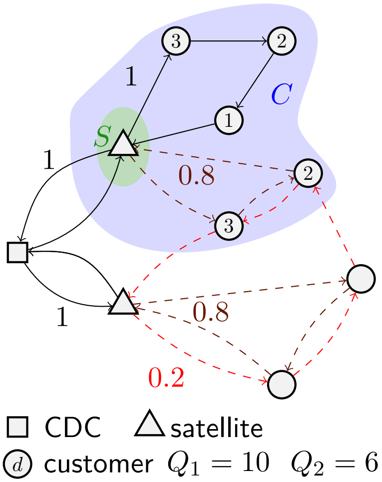
- variable y_e^s : nb of freighters from $s \in \mathcal{S}$ using $e \in E_2$
- data d_C : total demand of customers in $C \subset \mathcal{C}$
- variable b_S : freight passing through $S \subset \mathcal{S}$

Valid inequality

$$\sum_{s \notin S} \sum_{e \in \delta(C)} y_e^s \geq 2 \left\lceil \frac{d_C - b_S}{Q_2} \right\rceil$$

In the example : $0.4 \geq 2 \left\lceil \frac{11 - ?}{6} \right\rceil$

Satellites Supply Inequalities (SSI)



- variable y_e^s : nb of freighters from $s \in \mathcal{S}$ using $e \in E_2$
- data d_C : total demand of customers in $C \subset \mathcal{C}$
- variable b_S : freight passing through $S \subset \mathcal{S}$
- variable u_S : nb of urban trucks delivering S

Valid inequality

$$\sum_{s \notin S} \sum_{e \in \delta(C)} y_e^s \geq 2 \left\lceil \frac{d_C - b_S}{Q_2} \right\rceil$$

Upper bound : $b_S \leq Q_1 \lfloor u_S \rfloor$

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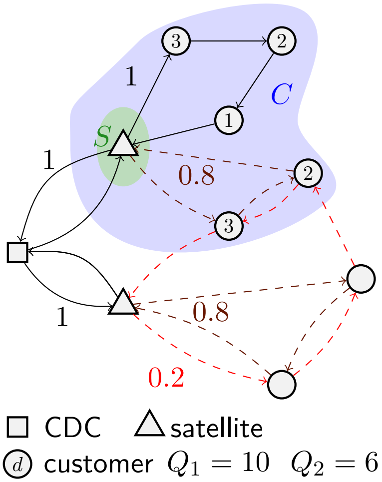

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Satellites Supply Inequalities (SSI)

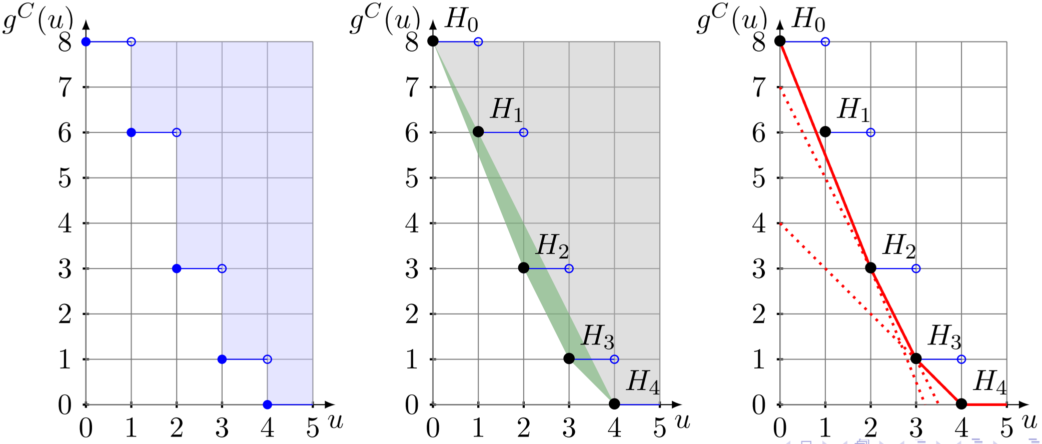
$$\sum_{s \notin S} \sum_{e \in \delta(C)} y_e^s \geq 2 \left\lceil \frac{d_C - Q_1 \lfloor u_S \rfloor}{Q_2} \right\rceil$$

In the example : $0.4 \geq 2 \left\lceil \frac{11 - 10}{6} \right\rceil$

Satellites Supply Inequalities (SSI)

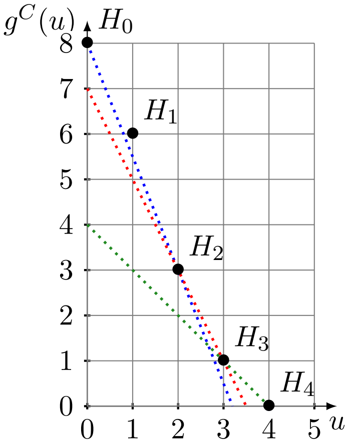
Linearization of the rhs g^C for a given C

$$\text{epi}\, h^C = \text{conv}(\{H_0, H_1, H_2, H_3, H_4\}) + \mathbb{R}^2_+$$



Satellites Supply Inequalities (SSI)

Cuts



Add following cuts :

$$\sum_{s \notin S} \sum_{i \in C} \sum_{j \notin S \cup C} y_{ij}^s + 5u_S \geq 16$$

$$\sum_{s \notin S} \sum_{i \in C} \sum_{j \notin S \cup C} y_{ij}^s + 4u_S \geq 14$$

$$\sum_{s \notin S} \sum_{i \in C} \sum_{j \notin S \cup C} y_{ij}^s + 2u_S \geq 8$$

Given a fractional solution $(\bar{x}, \bar{y}, \bar{u})$, look for the most violated inequality

$$\max_{\substack{S \subseteq \mathcal{S} \\ C \subseteq \mathcal{C}}} h^C(\bar{u}_S) - \underbrace{\sum_{s \notin S} \sum_{i \in C} \sum_{j \notin S \cup C} \bar{y}^s_{ij}}_{\text{cut on } G_2[\mathcal{C} \cup S^\complement] \text{ with value function } e \mapsto \sum_{s \in S^\complement} \bar{y}^s_e}$$

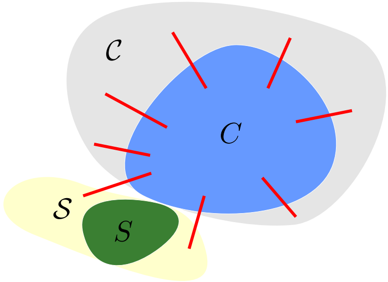


Figure: Simplified view

Heuristic overview

- **Step 1** Intelligent enumeration of subsets of satellites
- **Step 2** For each subset S picked: find cuts on subgraph $G_2[\mathcal{C} \cup S^\complement]$ that violate SSI

Visited satellite inequalities

- variable v_c^s : nb of city freighters visiting customer c from satellite s.
- variable $u_{\{s\}}$: nb of urban trucks delivering satellite s.

$$u_{\{s\}} \geq v_c^s, \quad c \in \mathcal{C}, \; s \in \mathcal{S}$$

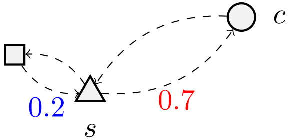


Figure: Example of violation.

Separation by enumeration.

BCP algorithm

- Use of VrpSolver : a generic BCP solver for VRPs [Pessoa et al., 2019]
  vrpsolver.math.u-bordeaux.fr

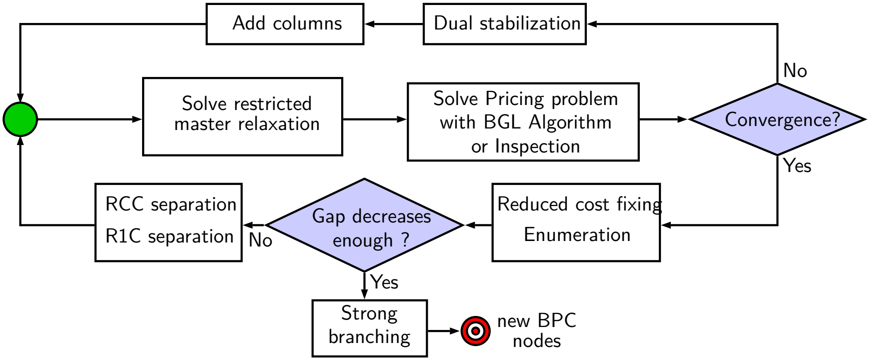
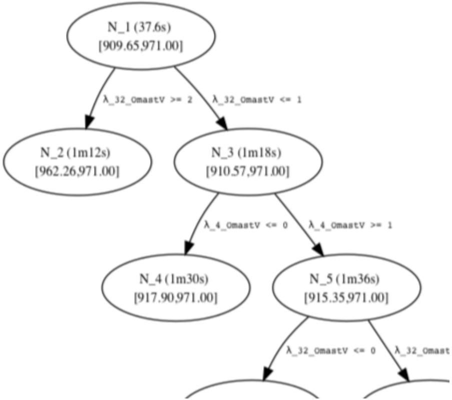


Figure: Solving a node

Branching rules

We branch on

- use of 1st level routes (variable λ)
- use of 2nd level edge by a tour from a given satellite (variable y)
- number of vehicles per satellites
- assignment of customers to satellites
- total number of city freighters used
- total number of urban trucks used



Branching rules

Branching on variables u_S.

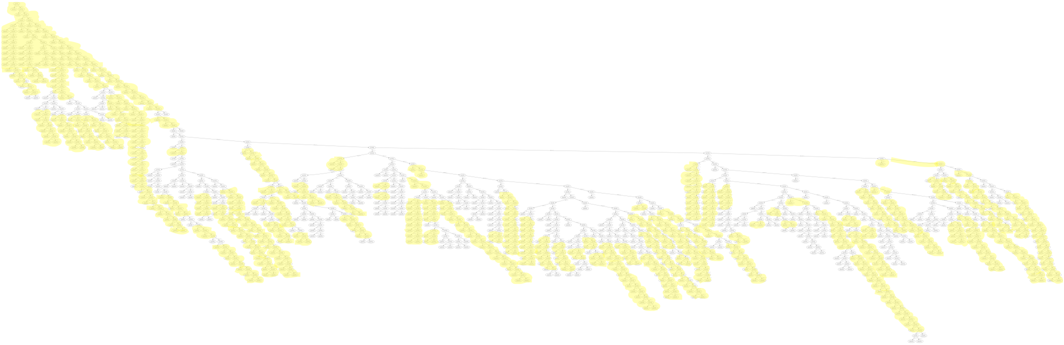


Figure: Branching tree of instance Set6A/A-n76-6.dat

1489 nodes. Time limit hit (3h).

Branching rules

Branching on variables u_S.

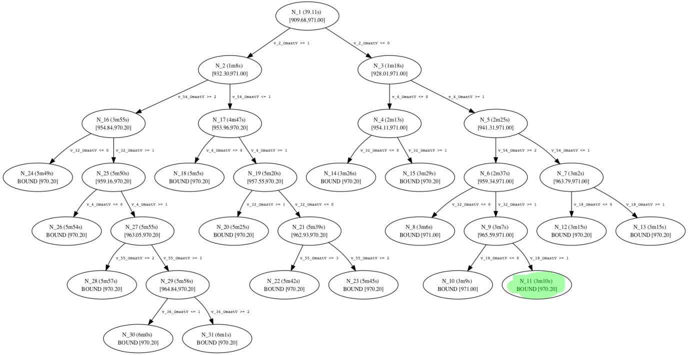


Figure: Branching tree of instance Set6A/A-n76-6.dat

31 nodes. Optimal solution found. (6mn 40)

Experiments

Experiment 1 - Effects of our contributions

- Time limit is 3h
- Best known solution as initial primal bound

Experiment 2 - Comparison with the state-of-the-art

- Time limit is 10h
- Search for primal bound with enumeration with false gap + Restricted Master Heuristic (Cplex 12.8 - Polishing heuristic [Rothberg, 2007] - Time limit is $|\mathcal{C}|/2.5$ sec. - Called at each node)

Instances of litterature

Set	nb of sats.	nb of cust.	Notes	Authors
4A	2, 3 or 5	50	$L_s < \|\mathcal{L}\|$	Crainic et al. 2010
4B	2, 3 or 5	50		Crainic et al. 2010
5	5 or 10	100 or 200		Hemmelmayr et al. 2012
6A	4, 5 or 6	50, 75 or 100		Baldacci et al. 2013
6B	4, 5 or 6	50, 75 or 100	$f_s \neq 0$	Baldacci et al. 2013

Table: Instances in literature

Experiment 1

Effects of our contributions

- **GVRS** is the generic VRPs solver without our contributions [Pessoa et al., 2019]
- **All** is GVRS + all our contributions

Variant	Root Gap (%)	Root Time (s)	Nodes	Time (s)	Solved
GVRS	4.29	83.6	76.0	2333.0	31/54
GVRS & branching u	4.28	99.5	24.5	1020.5	44/54
All & no Freight Cuts	0.68	177.8	5.9	421.4	49/54
All & no SSI	1.64	115.5	12.5	426.1	49/54
All & no VSI	0.71	238.6	6.6	501.0	50/54
All & no branching u	0.67	161.2	7.0	384.7	50/54
All	0.68	159.0	6.3	361.7	51/54

Experiment 2

GVRS vs. [Baldacci et al., 2013]

	GVRS				[Baldacci et al., 2013]	
Set	Root gap(%)	Nodes	t (s)	Solved	t (s)	Solved
4A	5.76	14.2	772	51/54	271	50/54
4B	4.45	12.7	550	52/54	232	52/54
5	5.83	222.9	20612	6/18	8405	3/6
6A	7.04	99.7	2604	24/27	802	22/27
6B	3.15	57.8	1562	24/27	513	19/27

Experiment 2

All vs. [Baldacci et al., 2013]

	All				[Baldacci et al., 2013]	
Set	Root gap(%)	Nodes	t (s)	Solved	t (s)	Solved
4A	0.91	3.3	144	54/54	271	50/54
4B	0.98	3.6	203	54/54	232	52/54
5	1.41	22.5	3215	15/18	8405	3/6
6A	0.89	4.9	233	27/27	802	22/27
6B	0.46	4.3	196	27/27	513	19/27

- We found three more optimal solutions with special parametrizations and time limit of 60h: `Set5/2eVRP100-5-1b`, `Set5/2eVRP200-10-1b`, and `Set5/2eVRP200-10-3b`
- We improved 10 primal solutions

Set7 - A set of 51 new instances

- **Goal :** reach the limits of our algorithm.
- Generated from instances for the CLRP [Schneider and Löffler, 2019]

Set	nb of sats.	nb of cust.
7	5, 10 or 15	100, 200 or 300

Table: New instances

- Time limit is 60h
- Feasible solution for 40/51 instances (up to 300 customers and 15 sats)
- Optimal solution for 23/51 instances (up to 300 customers and 10 sats)
- Only dual bound for 9 instances
- No dual bound for 2 instances

Conclusion

Paper : [Marques et al., 2019]

Our contributions

- New route based formulation
- Satellites Supply Inequalities
- Best exact method (and more generic than Baldacci et al.).
- Improved best known primal solutions
- Set of new instances
- Demo available on VrpSolver website

Ongoing work

- 2ECVRP with time-windows [Grangier et al., 2016] [Dellaert et al., 2018]

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