The Bin Packing and Vector Packing Problems: Two more VrpSolver applications.

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The Problem

Input

- Set T of items, set D of dimensions
- bin capacities Q^d ($d \in D$)
- item weights w_t^d ($t \in T, d \in D$)
- $D = \{1\}$ for Bin Packing
- $D = \{1, 2\}$ for (2D-Binary) Vector Packing

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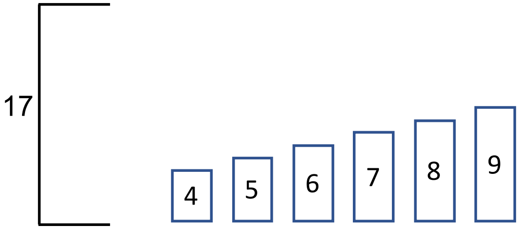
A Bin Packing Instance



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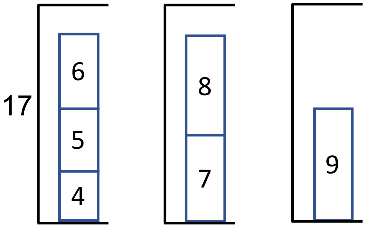

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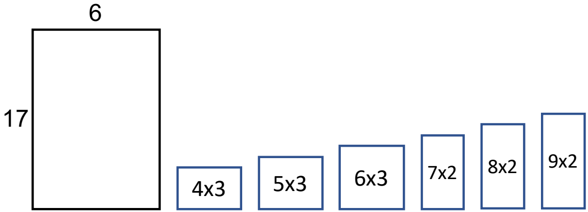

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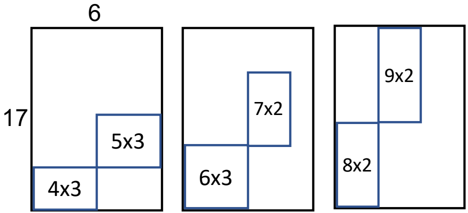

Decomposition

Input

- Set T of items, set D of dimensions
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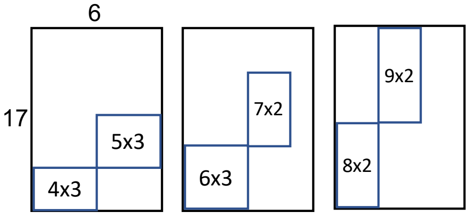


Formulation

h_t^p (constant): how many times item t is used in pattern $p \in P$

λ_p (variable): how many bins are filled with pattern p

$$\begin{aligned} \text{Min} \quad & \sum_{p \in P} \lambda_p && && \text{(1a)} \\ \text{S.t.} \quad & \sum_{p \in P} h_t^p \lambda_p \geq 1, && t = 1, \ldots, m, && \text{(1b)} \\ & \lambda_p \in \{0, 1\}, && p \in P. && \text{(1c)} \end{aligned}$$

P contains only feasible patterns
(p satisfying $\sum_{t \in T} w_t^d h_t^p \leq Q^d$, for all $d \in D$).

Relaxation

h_t^p (constant): how many times item t is used in pattern $p \in P$

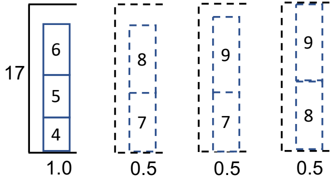
λ_p (variable): how many bins are filled with pattern p

$$\begin{aligned} \text{Min} \quad & \sum_{p \in P} \lambda_p && && \text{(2a)} \\ \text{S.t.} \quad & \sum_{p \in P} h_t^p \lambda_p \geq 1, && t = 1, \ldots, m, && \text{(2b)} \\ & \lambda_p \geq 0, && p \in P. && \text{(2c)} \end{aligned}$$

P contains only feasible patterns
(p satisfying $\sum_{t \in T} w_t^d h_t^p \leq Q^d$, for all $d \in D$).

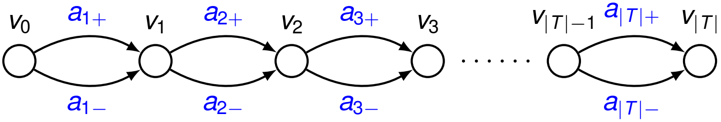
Relaxation: A Fractional Solution

$$\begin{aligned}
\text{Min} \quad & \lambda_{(4,5,6)} + \lambda_{(7,8)} + \lambda_{(7,9)} + \lambda_{(9,8)} + \cdots && \text{(3a)}\\
\text{S.t.} \quad & \lambda_{(4,5,6)} + \lambda_{(4)} + \cdots \geq 1, && \text{(3b)}\\
& \lambda_{(4,5,6)} + \lambda_{(5)} + \cdots \geq 1, && \text{(3c)}\\
& \lambda_{(4,5,6)} + \lambda_{(6)} + \cdots \geq 1, && \text{(3d)}\\
& \lambda_{(7,8)} + \lambda_{(7,9)} + \cdots \geq 1, && \text{(3e)}\\
& \lambda_{(7,8)} + \lambda_{(8,9)} + \cdots \geq 1, && \text{(3f)}\\
& \lambda_{(7,9)} + \lambda_{(8,9)} + \cdots \geq 1, && \text{(3g)}\\
& \lambda_p \geq 0, \qquad p \in P. && \text{(3h)}
\end{aligned}$$



The VrpSolver Model

Graph G



- Resources $d \in D = R = R^M$ with consumption: $q_{a_{t+},d} = w_t^d$, $q_{a_{j-},d} = 0$, $t \in T, d \in D$
- Consumption bounds: $[l_{v_j,d}, u_{v_j,d}] = [0, Q^d]$, $t \in T$.
- Same as [Hessler et al., 2018] and [Wei et al., 2019].

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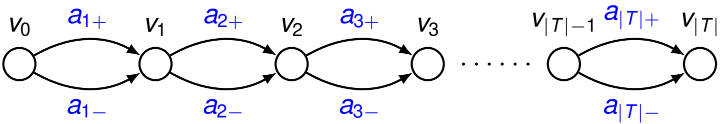

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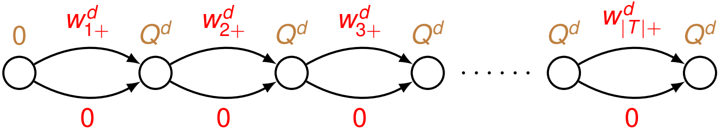
RCSP Subproblem



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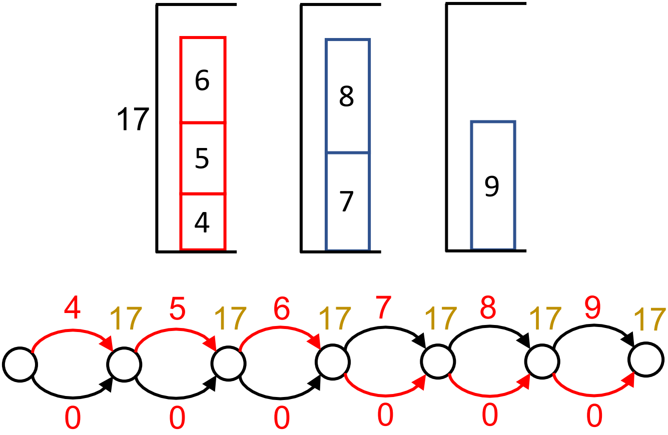

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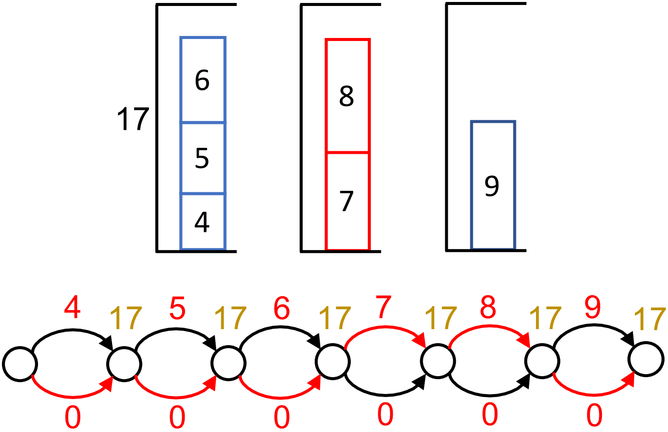

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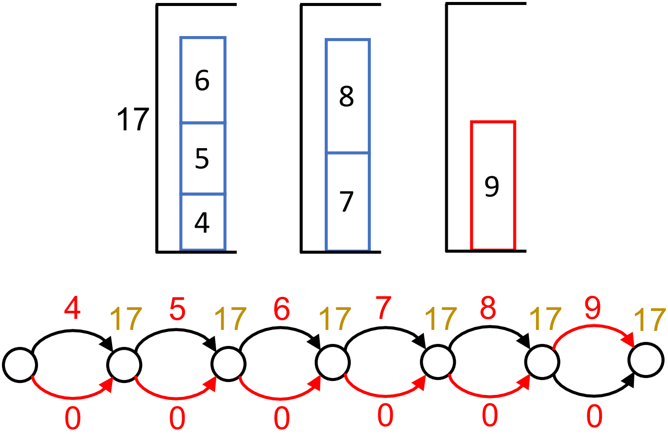

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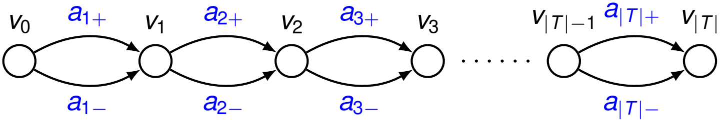
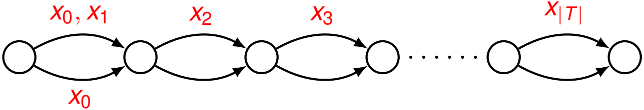


- Mapping: $M(x_0) = \{a_{1+}, a_{1-}\}$, $M(x_t) = \{a_{t+}\}, t \in T$

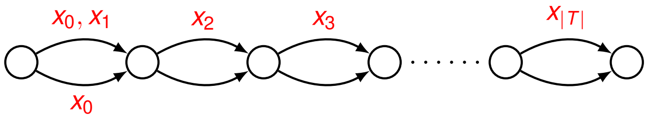
The VrpSolver Model

Arc mappings



The VrpSolver Model

Arc mappings



Formulation

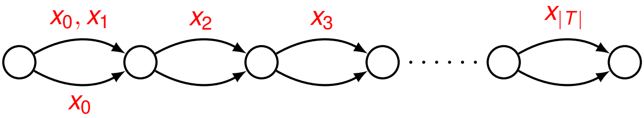
$$\begin{aligned} \text{Min} \quad & x_0 \\ \text{S.t.} \quad & x_t \geq 1, \qquad t \in T; \end{aligned}$$

Additional Elements

- Subproblem cardinality: $L = 0$, $U = \infty$
- Packing sets: $\mathcal{B} = \cup_{t \in T}\{\{a_{t+}\}\}$
- Branching over accumulated resource consumption and, if still needed, by Ryan and Foster rule
- Enumeration is on

The VrpSolver Model

Arc mappings



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- Enumeration is on

Advanced Techniques borrowed from Vehicle Routing

- Bucket graph-based labelling algorithm for the RCSP pricing [Righini and Salani, 2006] [Sadykov et al., 2017]
- Automatic dual price smoothing stabilization [Wentges, 1997] [Pessoa et al., 2017]
- Reduced cost fixing of (bucket) arcs in the pricing problem [Ibaraki and Nakamura, 1994] [Irnich et al., 2010] [Sadykov et al., 2017]
- Limited-Memory Rank-1 Cuts [Jepsen et al., 2008] [Pecin et al., 2017b] [Pecin et al., 2017c] [Pecin et al., 2017a]
- Enumeration of elementary routes [Baldacci et al., 2008]
- Multi-phase strong branching [Pecin et al., 2017b]
- Generic (strong) diving heuristic [Sadykov et al., 2018]

Bin Packing: computational results

- Comparison with the best BCP [Wei et al., 2019] on AI and ANI instances by [Delorme et al., 2016].
- All instances by [Falkenauer, 1996] (up to 501 items) and by [Schoenfield, 2002] (up to 200 items, Hard28) solved in up to 3m37s (16s in the average) but pseudopolynomial formulations lead to better results.
- Initial upper bound is rounded-up lower bound plus one (easy for heuristics).
- Diving heuristic not enabled for ANI instances.

Bin Packing: computational results

Instance class	$\lvert J \rvert$	Best BCP		Our BCP	
		N	T	N	T
ANI200	201	50/50	14s	50/50	17s
ANI400	402	47/50	>7m16s	50/50	1m36s
ANI600	600	0/50	>1h	3/50	>58m
ANI800	801	0/50	>1h	0/50	>1h
AI200	202	50/50	4s	50/50	52s
AI400	403	46/50	>6m	46/50	>8m11s
AI600	601	27/50	>29m	35/50	>24m
AI800	802	15/50	>46m	26/50	>46m

- Best BCP: [Wei, Luo, Baldacci, and Lim, 2019]

Vertex Packing: computational results

Comparison with the state-of-the-art on the harderst 2-resources 200-items instances by [Caprara and Toth, 2001]

Algorithm	Class 1		Class 4	
	N	T	N	T
[Brandão and Pedroso, 2016]	10/10	2h07m	0/10	>2h
[Hu et al., 2017]	0/10	>10m	0/10	>10m
[Hessler et al., 2018]	3/10	>47m	0/10	>1h
Our BCP	10/10	2m42s	10/10	2m33s
	Class 5		Class 9	
[Brandão and Pedroso, 2016]	0/10	>2h	0/10	>2h
[Hu et al., 2017]	7/10	>6m	0/10	>10m
[Hessler et al., 2018]	7/10	>41m	0/10	>1h
Our BCP	10/10	12m10s	8/10	>27m

New Branching Scheme for Bin Packing

VrpSolver Model

- Branching over accumulated resource consumption and, if still needed, by Ryan and Foster rule

New Branching Scheme for Bin Packing

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- Branching over accumulated (disposable) resource consumption and, if still needed, by Ryan and Foster rule (never needed)

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VrpSolver Model

- Branching over accumulated (disposable) resource consumption and, if still needed, by Ryan and Foster rule (never needed)
- Advantages: keeps the princing structure (robust), allows stronger dominance rule.
- Disadvantage: subproblem solutions with positive capacity slacks may be feasible in both branches.

New Branching Scheme for Bin Packing

VrpSolver Model

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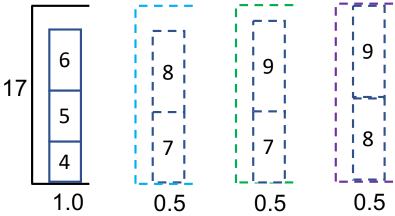
Example



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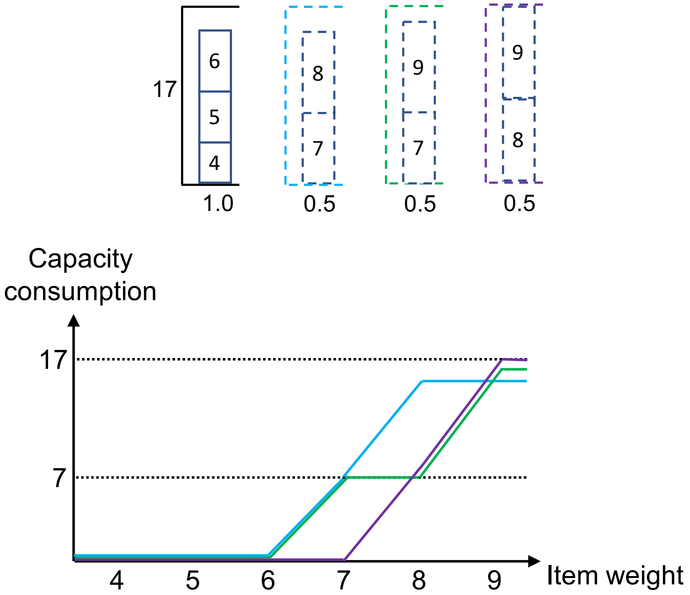

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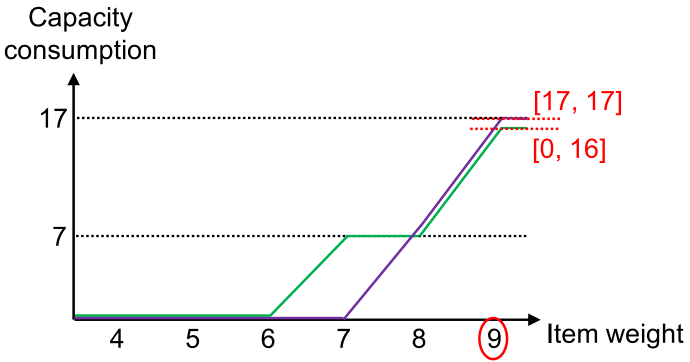
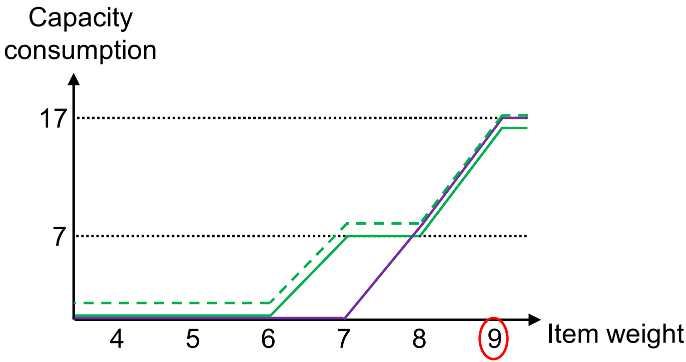


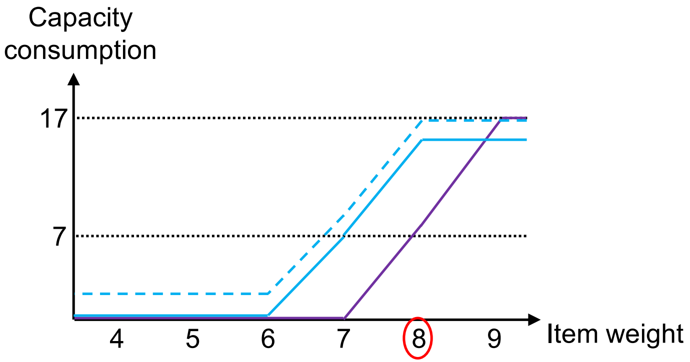
- Resource consumption bounds for branching on item 9 are $[0, 16]$ and $[17, 17]$
- Both paths can meet bounds $[17, 17]$ (disposable resources)

New Branching Scheme for Bin Packing



- Must consider both minimum and maximum resource consumptions
- No effective branching is possible for item 9

New Branching Scheme for Bin Packing



- Item 8 admits an effective branching

New Branching Scheme for Bin Packing

The scheme

- In an effective branching, for each branch, there is at least one non-zero λ variable whose path becomes infeasible.
- Some λ variables may still have paths feasible for both branches.
- Previously proposed in [Gélinas et al., 1995] for vehicle routing.

New Branching Scheme for Bin Packing

The scheme

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Theorem

- For Bin Packing, any fractional solution with no effective branching has the same projection as some convex combination of integer solutions in the arcs space.

New Branching Scheme for Bin Packing

Theorem

- Any fractional solution with no effective branching has the same projection as some convex combination of integer solutions in the arcs space.

Sketch of the proof

Step 1 Any fractional solution obtained with tight (disposable) resource consumption bounds has the same projection as some convex combination of integer solutions in the arcs space.

- The master problem relaxation is equivalent to a minimum cost flow problem.

New Branching Scheme for Bin Packing

Theorem

- Any fractional solution with no effective branching has the same projection as some convex combination of integer solutions in the arcs space.

Sketch of the proof

Step 2 Any fractional solution with no effective branching is feasible for some set of tight resource consumption bounds (and the theorem follows from Step 1).

- No effective branching $\Rightarrow$ for every item t, and consumption threshold q, every path is feasible either under consumption bound $\leq q - 1$ or $\geq q$.
- Let q^* be the maximum threshold the makes all paths feasible for consumption bound $\geq q^*$.
- For threshold $q^* + 1$ all paths are feasible for consumption bound $\leq q^*$.
- Thus, all paths are feasible for the tight consumption range $[q^*, q^*]$.

Conclusions and Future Work

Conclusions

- VrpSolver is an useful tool for testing (existing) advanced BCP techniques on new problems
- For Bin Packing, it is competitive, and for Vector Packing, superior to the state-of-the art.
- Such tests may inspire interesting new investigations (e.g. the new branching scheme)

Possible Extensions to the New Branching Scheme

- Bin Packing with Multiple Size Bins
- Cutting Stock
- Generic VrpSolver Model (multiple resources)

THANKS!

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