An Exact Solution Framework for Multi-Trip Vehicle Routing Problems with Time Windows

Autumn School on Advanced BCP Tools

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Introduction

- Most of the literature on the Vehicle Routing Problem (VRP) addresses problems where each vehicle can perform at most one trip per day
- Many contributions on VRPs where vehicles can perform multiple trips have been published in the last decade
- These problems are called Multi-Trip Vehicle Routing Problems (MTVRP)

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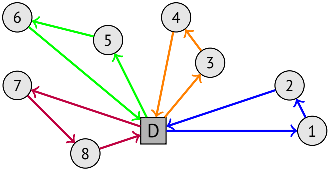

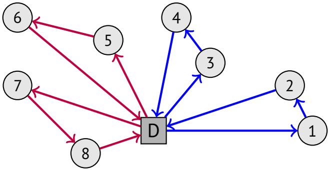

Motivation

- Such an increasing interest in MTVRPs is due to new practices in, e.g., city logistics and last-mile delivery
- The need of limiting noise and pollution in city centers requires the usage of small vans, electric vehicles, and/or drones and forbids large trucks from entering city centers
- The limited capacity/autonomy of these vehicles forces them to perform multiple trips and to return to the depot to reload multiple times over the day

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Main Research Question to Address in this Talk

What is the best model to solve an MTVRP (with side constraints) to optimality?

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Based on the state-of-the-art exact methods for lots of VRPs...

Set Partitioning Models!

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Definition of the Multi-Trip VRP I

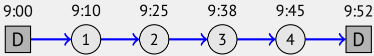
Input Data

- N set of customers
- V vertex set, $V = N \cup \{0\}$, where 0 is the depot
- $\mathcal{A}$ arc set, $\mathcal{A} = \{(i,j) \mid i,j \in V : i \neq j\}$
- $\mathcal{G}$ directed graph, $\mathcal{G} = (V, \mathcal{A})$
- t_{ij} travel time of arc $(i,j) \in \mathcal{A}$
- K fleet of identical capacitated vehicles, $|K| = m$
- q_i demand of customer $i \in N$
- Q vehicle capacity
- T length of the planning horizon

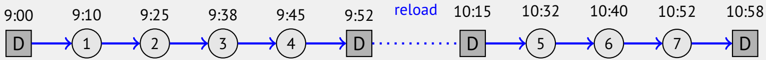
Definition of the Multi-Trip VRP II

Definitions

- A trip is a sequence of customers, whose total demand does not exceed Q, that can be visited by a vehicle in between two visits at the depot, and that has a fixed departure time from the depot



- A journey is a sequence of non-overlapping trips assigned to a vehicle whose total travel time does not exceed T



The MTVRP aims at defining a set of at most m journeys such that:

1. each customer is visited exactly once
2. the total traveled time is minimized

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Models with 3- and 4-index Variables

4-index Variables

$x_{ij}^{kh} \in \{0,1\}$ equal to 1 if trip h of vehicle $k \in K$ traverses arc $(i,j) \in \mathcal{A}$ (0 otherwise)

3-index Variables with Vehicle Index (without Trip Index)

$x_{ij}^{k} \in \{0,1\}$ equal to 1 if vehicle $k \in K$ traverses arc $(i,j) \in \mathcal{A}$ (0 otherwise)

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Models with 3- and 4-index Variables

Pros and Cons

- Polynomial number of variables
- Can be solved with commercial solvers
- Easy to embed additional side constraints

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- High integrality gaps
- BigM constraints
- Symmetries in the vehicles

2-Index Arc-based Model (Koc and Karaoglan (2011)) I

Variables

$x_{ij} \in \{0,1\}$ equal to 1 if arc $(i,j) \in \mathcal{A}$ is traversed (0 otherwise)

$x'_{ij} \in \{0,1\}$ equal to 1 if a vehicle visits customers $i,j \in N$ $(i \neq j)$ consecutively with a stop at the depot in between (0 otherwise)

$\ell_i \in \mathbb{R}_+$ load on board after visiting customer $i \in N$

$a_i \in \mathbb{R}_+$ arrival time at customer $i \in N$

2-Index Arc-based Model (Koc and Karaoglan (2011)) II

$\min \sum_{(i,j)\in\mathcal{A}} t_{ij}x_{ij}$		[Minimize travel times]	(1a)
s.t. $\sum_{(i,j)\in\mathcal{A}} x_{ij} = 1$	$i \in N$	[Serve each customer]	(1b)
$\sum_{(i,j)\in\mathcal{A}} x_{ij} = \sum_{(j,i)\in\mathcal{A}} x_{ji}$	$i \in V$	[Flow conservation]	(1c)
$\ell_i + q_j \leq \ell_j + Q(1 - x_{ij})$	$i \in N\ j \in V$	[Subtour + Load on board]	(1d)
$a_i + t_{ij} \leq a_j + T(1 - x_{ij})$	$i \in V\ j \in N$	[Subtour + Arrival time]	(1e)
$a_i + (t_{i0} + t_{0j}) \leq a_j + T(1 - x'_{ij})$	$i, j \in N : i \neq j$	[Arrival time depot visit]	(1f)
$t_{0i} \leq a_i \leq T - t_{i0}$	$i \in N$	[Planning horizon]	(1g)
$\sum_{j\in N} x'_{ij} \leq x_{i0}$	$i \in N$	[Link x with x']	(1h)
$\sum_{j\in N} x'_{ij} \leq x_{0j}$	$j \in N$	[Link x with x']	(1i)
$\sum_{(0,j)\in\mathcal{A}} x_{0j} - \sum_{i,j\in N : i\neq j} x'_{ij} \leq m$		[Number of vehicles]	(1j)
$x_{ij} \in \{0, 1\}$	$(i, j) \in \mathcal{A}$		(1k)
$x'_{ij} \in \{0, 1\}$	$i, j \in N : i \neq j$		(1l)
$q_i \leq \ell_i \leq Q, \quad a_i \in \mathbb{R}_+$	$i \in N$		(1m)

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- BigM constraints
- Instances with 50 customers are already difficult to close

Trip-based Model (Mingozzi, Roberti, and Toth (2013))

$\mathcal{H}$ set of all feasible trips

c_h cost of trip $h \in \mathcal{H}$

α_{ih} trip $h \in \mathcal{H}$ serves customer $i \in N$ ($\alpha_{ih} = 1$) or not ($\alpha_{ih} = 0$)

d_h duration of trip $h \in \mathcal{H}$

Variables

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$$\sum_{h \in \mathcal{H}} d_h x_{hk} \leq T \qquad k \in K \quad \text{[Planning horizon]} \tag{2c}$$

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Pros and Cons

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- Instances with 100-120 customers can be closed
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- Small integrality gaps (smaller than trip-based model)
- Instances with 100-120 customers can be closed
- Easy to embed additional side constraints

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- Exponential number of variables
- Column generation/branch(and-cut)-and-price needed
- Pricing problem more difficult than trip-based model
- Additional constraints can make the pricing problem (even more) difficult

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Main Side Constraints and Academic Extensions

- Time Windows: each customer $i \in N$ must be visited within a time interval $[a_i, b_i]$
- Service-Dependent Loading Times: vehicle loading time at the depot depends on the customers visited in the next trip
- Limited Trip Duration: maximum time between the departure from the depot and the arrival time at the last customer of the trip
- Profits: a profit p_i is associated with each customer $i \in N$; hierarchical objective function: maximize profit first; minimize routing cost second

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Main Side Constraints and Academic Extensions

Reference	Time Windows	Service-Dependent Loading Times	Limited Trip Duration	Profits
Exact Methods				
Azi, Gendreau, and Potvin (2010)	✓	✓	✓	✓
Macedo et al. (2011)	✓	✓	✓	✓
Hernandez et al. (2014)	✓	✓	✓	
Hernandez et al. (2016)	✓	✓		
Heuristic Methods				
Azi, Gendreau, and Potvin (2014)	✓	✓	✓	✓
Wang, Liang, and Hu (2014)	✓	✓	✓	✓
Cattaruzza, Absi, and Feillet (2016a)	✓	✓		
Anaya-Arenas et al. (2016)	✓		✓	

From Cattaruzza, Absi, and Feillet (2016b)

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τ_{th} trip $h \in \mathcal{H}$ is active at time $t \in [a_0, b_0]$ ($\tau_{th} = 1$) or not ($\tau_{th} = 0$)

Variables

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$$\sum_{h \in \mathcal{H}} \tau_{th} x_h \leq m \qquad t \in [a_0, b_0] \qquad \text{[No overlaps]} \tag{4c}$$

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- Exponential number of variables
- Column generation/branch(-and-cut)-and-price needed
- Side constraints make the pricing problem difficult
- Constraints (4c) to add in a cutting plane fashion
- Instances with 25 customers can be out of reach

Journey-based Model (Hernandez et al. (2014, 2016))

$\mathcal{R}$ set of all feasible journeys

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- Easy to embed additional side constraints both related to trips and journeys

Journey-based Model (Hernandez et al. (2014, 2016))

Pros and Cons

- Small integrality gaps (smaller than trip-based model)
- Easy to embed additional side constraints both related to trips and journeys

- Exponential number of variables
- Column generation/branch(and-cut)-and-price needed
- Pricing problem more difficult than trip-based model
- Instances with 25 customers can be out of reach

The Concept of Structure

Definition of Structure

A structure $s = (0, i_1, i_2, \ldots, i_{\mu_s}, 0)$ is an ordered set of μ_s customers that can be visited in between two visits at the depot and can start from the depot within time interval $[e_s, \ell_s]$, such that:

1. capacity constraints are satisfied
2. the duration d_s and the cost c_s are constant for each departure time from the depot within $[e_s, \ell_s]$
3. the duration d_s is the minimum duration to serve the set of customers in the given order

Structure-based Model (Paradiso et al. (2019))

$\mathcal{S}$ set of all feasible structures

c_s cost of structure $s \in \mathcal{S}$

α_{is} structure $s \in \mathcal{S}$ serves $i \in N$ ($\alpha_{is} = 1$) or not ($\alpha_{is} = 0$)

Variables

$x_s \in \{0, 1\}$ structure $s \in \mathcal{S}$ is selected ($x_s = 1$) or not ($x_s = 0$)

$$\min \sum_{s \in \mathcal{S}} c_s x_s \quad \text{[Minimize travel costs]} \tag{6a}$$

$$\text{s.t.} \sum_{s \in \mathcal{S}} \alpha_{is} x_s = 1 \quad i \in N \quad \text{[Serve each customer]} \tag{6b}$$

$$\sum_{s \in \widehat{\mathcal{S}}} x_s \leq \eta_m(\widehat{\mathcal{S}}) \quad \widehat{\mathcal{S}} \subseteq \mathcal{S} \quad \text{[Structure feasibility constraints]} \tag{6c}$$

$$x_s \in \{0, 1\} \quad s \in \mathcal{S} \tag{6d}$$

where $\eta_m(\widehat{\mathcal{S}})$ is the maximum number of structures of the set $\widehat{\mathcal{S}}$ that can be simultaneously in a solution given the number of vehicles m

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- Small integrality gaps
- Easy to embed additional side constraints related to trips
- Fewer variables than trip-based (and journey-based) model

- Exponential number of variables
- Column generation/branch(-and-cut)-and-price needed
- Constraints (6c) to add in a cutting plane fashion

Trip vs Journey vs Structure (-based Models)

	Trip	Journey	Structure
Integrality gap	👍	👍👍	👍(👍)
Number of variables	👎👎	👎👎👎	👎
Number of constraints	👎	👍	👎👎
Trip-related constraints	👍	👍	👍
Journey-related constraints	👎	👍	👎
Complexity of algorithms	👎	👎	👎

Sketch of an Exact Method based on Structure-based Model

Paradiso et al. (2019)

1. Compute SP Bound: solve LP relaxation of (7) without (7c) to compute dual sol. $\boldsymbol{u}^1$ of cost LB_1
2. Enumerate Structures: enumerate structures ($\tilde{\mathcal{S}}$) of red. cost $\leq \mathrm{UB} - \mathrm{LB}_1$ w.r.t. $\boldsymbol{u}^1$, where UB is a guessed upper bound
3. Compute SP plus Relaxed SFC: solve LP relaxation of (7) with relaxed (7c) to compute dual sol. $\boldsymbol{u}^2$ of cost LB_2
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Group	$\|N\|$	Inst	%Gap	Opt	T_{tot}	%Gap	Opt	T_{tot}	%Gap	Opt	T_{tot}
C	25	8	2.24	8	108	2.12	7	805	0.73	8	19
R	25	11	2.41	11	646	1.19	7	6,925	0.78	11	115
RC	25	8	5.41	6	6,671	2.86	5	2,963	1.91	8	880
C	40	8							1.51	7	2,170
R	40	11							0.41	10	418
RC	40	8							0.83	8	872
C	50	8							1.41	3	3,577
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C	25	16	1.91	16	420	0.38	16	14
R	25	22	0.76	22	33	0.25	22	2
RC	25	16	2.35	11	18	0.49	16	2
C	40	16	1.25	13	511	0.48	16	151
R	40	19	1.43	12	1,738	1.06	19	220
RC	40	2	-	0	-	0.67	2	11
C	50	16				0.22	16	62
R	50	22				0.22	22	20
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$\|N\|$	Inst	%Gap	Opt	T_{tot}	%Gap	Opt	T_{tot}
10	10	4.49	10	0	0.40	10	0
15	10	5.47	4	9	1.28	10	1
20	10	3.69	5	18	0.86	10	2
25	37	2.68	22	59	0.62	37	1
30	10		0		0.52	10	4
35	10		0		0.44	10	11
40	37	3.83	4	4,168	0.20	37	5
45	10		0		0.36	10	13
50	5		0		1.72	5	275

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25	37	2.68	22	59	0.62	37	1
30	10		0		0.52	10	4
35	10		0		0.44	10	11
40	37	3.83	4	4,168	0.20	37	5
45	10		0		0.36	10	13
50	5		0		1.72	5	275

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Conclusions

- Increasing interest in MTVRPs, mainly motivated by city logistics and last-mile delivery
- Trip-based and journey-based models are effective to solve the MTVRP
- To handle side constraints, structure-based models seem the better choice, even better than set-partitioning models

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Open Questions

- Research on MTVRPs is scarce and 50-customer instances are already challenging, how can large instances be solved?
- Are there better models (maybe models not based on arcs, structures, trips, or journeys)?
- Can we use models not based on arcs or routes to solve other VRPs?

R. Paradiso, R. Roberti, D. Laganá, W. Dullaert. An Exact Solution Framework for Multi-Trip Vehicle Routing Problems with Time Windows. *Operations Research* (forthcoming)

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